Quadratische Fun m. Parameter

1.0 $f_a(x) = ax^2 - 4x + 2 \quad ; \quad a \in \mathbb{R} \setminus \{0\}$

1.1 $a > 0$: Nach oben offene Parabeln ; Gemeinsam $S_y(0|2)$
$a < 0$ „ unten „ „

1.2 $f_a(x) = a\left(x^2 - \frac{4}{a}x + \left(\frac{2}{a}\right)^2 - \left(\frac{2}{a}\right)^2 + \frac{2}{a}\right) = a\left(x - \frac{2}{a}\right)^2 - \frac{4}{a} + 2$

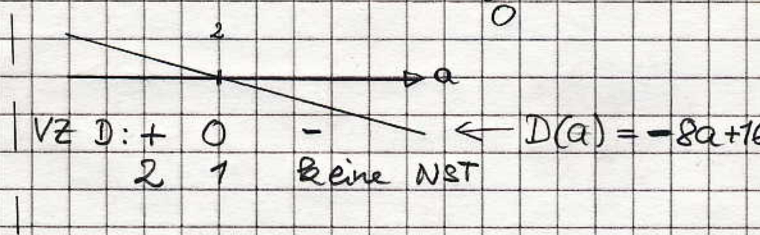
$= a\left(x - \frac{2}{a}\right)^2 + \frac{2a}{a} - \frac{4}{a} \Rightarrow S\left(\frac{2}{a} \,\middle|\, \frac{2a-4}{a}\right)$

[Für $a \to +\infty$: $x_S \to 0^+$; $y_S \to 2^-$ weil $2 - \underbrace{\frac{4}{a}}_{\to 0} \Rightarrow S \to S_y$

1.3 $ax^2 - 4x + 2 = 0$

$D = 16 - 4 \cdot a \cdot 2 = 16 - 8a$

$16 - 8a = 0 \Leftrightarrow a = 2$



• $a < 2$; 2 NST

$x_{1/2} = \frac{1}{2a}\left(4 \pm \sqrt{16 - 8a}\right) = \frac{1}{2a}\left(4 \pm \sqrt{4(4-2a)}\right) = \frac{2}{2a}\left(2 \pm \sqrt{4 - 2a}\right)$

• $a = 2$: 1 NST ; Scheitel

$x_{1/2} = x_S = -\frac{b}{2a} = \frac{4}{2a} = \frac{2}{a}$

• $a > 2$: keine NST

1.4 $P(0|0)$ in $f_a(x)$: $a \cdot 0 - 4 \cdot 0 + 2 = 0 \Leftrightarrow 2 = 0$ (f) $\Rightarrow$ Es gibt kein passend. a

(War auch klar : $S_y(0|2)$ unabh. von a)

1.5 $a = 2$: $N_{1/2}\left(\frac{2}{a} \,|\, 0\right) = N(1|0) = S\left(\frac{2}{2} \,\middle|\, \frac{2 \cdot 2 - 4}{2}\right) = S(1|0)$

1.6 $f_2(x) = 2x^2 - 4x + 2 = 2(x-1)^2$; BiFo!

$2x^2 - 4x + 2 \leq 8 \;|:2$; NR : $x^2 - 2x + 1 = 4 \Leftrightarrow x^2 - 2x - 3 = 0$

$\Leftrightarrow (x+1)(x-3) = 0$ $\quad 1 \cdot (-3) = -3$, $\; 1 - 3 = -2$

$x_1 = -1 \qquad x_2 = 3$

VZ + 0 − 0 +

$\mathbb{L} = [-1 ; 3]$

Bemerkung :
„Nicht übersteigen"
⇝ 8 ist als y-Wert mit dabei

oder mit Linearfaktoren :

x		−1		3	
$(x+1)$	−	0	+	+	+
$(x-3)$	−	−	−	0	+
Ges	+	0	−	0	+

2. Für alle Aufgaben: keine NST $\Leftrightarrow D < 0$

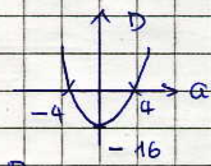
a) $D = 4 - 4 \cdot a \cdot 1 = -4a + 4$; $D = 0 \Leftrightarrow a = 1 \Rightarrow D < 0 \Leftrightarrow \underline{a > 1}$

b) $D = 1 - 4(-1)(a-1) = 1 + 4a - 4 = 4a - 3$

$4a - 3 < 0 \Leftrightarrow \underline{a < \frac{4}{3}}$

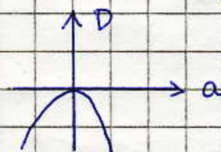
c) $D = a^2 - 4 \cdot 2 \cdot 2 = a^2 - 16 = (a+4)(a-4)$

$D < 0 \Leftrightarrow \underline{a \in]-4;4[}$



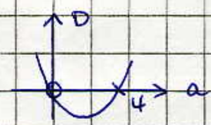
d) $D = (-a)^2 - 4 \cdot a \cdot 2a = a^2 - 8a^2 = -7a^2$

$D < 0 \Leftrightarrow \underline{a \in \mathbb{R} \setminus \{0\}}$



e) $D = a^2 - 4 \cdot (-1) \cdot (-a) = a^2 - 4a = a(a-4)$

$D < 0 \Leftrightarrow \underline{a \in]0;4[}$



f) $D = (-a)^2 - 4 \cdot 3(-a-1) = a^2 + 12a + 12 \stackrel{!}{=} 0$; $a_{1/2} = \frac{1}{2}\left(-12 \pm \sqrt{12^2 - 4 \cdot 12}\right)$

$D < 0 \Leftrightarrow \underline{a \in]-6-2\sqrt{6}\,;\,-6+2\sqrt{6}[}$ $\qquad a_{1/2} = \frac{1}{2}\left(-12 \pm \sqrt{96}\right)$

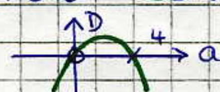
g) $D = (a+1)^2 - 4 \cdot (-2) \cdot (a-5) = a^2 + 2a + 1 + 8a - 40 = a^2 + 10a - 41$

$a_{1/2} = \frac{1}{2}\left(-10 \pm \sqrt{100 - 4 \cdot 1 \cdot (-41)}\right) = \frac{1}{2}\left(-10 \pm \sqrt{264}\right)$

$\underline{a \in]-5-\sqrt{66}\,;\,-5+\sqrt{66}[}$

h) $D = a^2 - 4 \cdot a(a-3) = a^2 - 4a^2 + 12a = -3a^2 + 12a = -3a(a-4)$

$\underline{a \in \mathbb{R} \setminus [0;4]} =]-\infty;0[\cup]4;\infty[$



3. $f_t(x) = (t^2-1)\left[x^2 + \frac{2(1-t^2)}{(t^2-1)} + \frac{2t}{t^2-1}\right]$; $\frac{2(1-t^2)}{t^2-1} = \frac{-2(t^2-1)}{t^2-1}$

$= (t^2-1)\left[x^2 - 2x + 1^2 - 1^2 + \frac{2t}{t^2-1}\right]$

$= (t^2-1)\left[(x-1)^2 - 1 + \frac{2t}{t^2-1}\right]$

$= (t^2-1)(x-1)^2 - 1(t^2-1) + 2t$

$= (t^2-1)(x-1)^2 - t^2 + 2t + 1$

keine BiFo

$\underline{S(1 \mid -t^2 + 2t + 1)}$

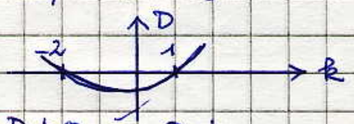
4.0 $f_k(x) = -\frac{1}{4}x^2 - kx + k - 2 \; ; \; k \in \mathbb{R}$

4.1 $f_k(x) = -\frac{1}{4}\left(x^2 + 4k + (2k)^2 - (2k)^2\right) + k - 2$ $\quad -\frac{1}{4}\cdot(4k^2)$

$= -\frac{1}{4}(x+2k)^2 + k^2 + k - 2$; (*) $S(-2k \mid k^2 + k - 2)$

4.2 $D = k^2 - 4\cdot(-\frac{1}{4})(k-2) = k^2 + k - 2 = (k+2)(k-1)$ $\quad -1\cdot 2 = -2$, $2 - 1 = 1$

$k_1 = -2 \quad k_2 = 1$



VZD +0 − 0 +

- $k \in]-2; 1[$: keine NST
- $k \in \{-2; 1\}$: eine NST

$x_{1/2} = -\frac{b}{2a} = -\frac{-k}{-\frac{1}{4}\cdot 2} = -2k$

$k = -2$: $N_{-2}(4 \mid 0)$

$k = 1$: $N_1(-2 \mid 0)$

- $k \in \mathbb{R} \setminus [-2; 1]$

$x_{1/2} = \frac{1}{2\cdot(-\frac{1}{4})}\left(k \pm \sqrt{k^2 + k - 2}\right) = -2k \pm 2\sqrt{k^2 + k - 2}$; $N_{1/2}(x_{1/2} \mid 0)$

4.3 $f_0(x) \overset{(*)}{=} f_k(x)$ (Vgl.: Bestimmung d. Büschelpktes v. Geraden)

$\Leftrightarrow -\frac{1}{4}x^2 - 2 = -\frac{1}{4}x^2 - kx + k - 2$

Logo: $f_0(x) = f_0(x)$

$\Leftrightarrow \quad 0 = -kx + k \Leftrightarrow kx = k \quad | : k$

1. Fall : $k = 0$: $0x = 0$ (w) ; ∞ viele SP ; Parabeln identisch

2 Fall : $k \neq 0$: $x_{SP} = 1$ unabh. vom Parameter d.h. für alle Parameter-Werte

u $\quad y_{SP} = f_0(x_{SP}) = -\frac{1}{4}\cdot 1^2 - 2 = -\frac{9}{4}$

(*) Probiere auch: $k = 2$

Gemeinsamer Punkt $G(1 \mid -\frac{9}{4})$

4.4 $k = -3$: $S_{-3}(-2\cdot(-3) \mid (-3)^2 + (-3) - 2) = S_{-3}(6 \mid 4)$ (4.1)

$k = -1$: $S_{-1}(2 \mid -2)$

$k = 0$: $S_0(0 \mid -2)$

$k = 1$: $S_1(-2 \mid 0)$ ($\rightarrow$ 4.2)

$k = 2$: $S_2(-4 \mid 8)$

vom Scheitel aus

zeichnen!

5.0 $f_t(x) = 2x^2 + tx + 2 \;;\; t \in \mathbb{R}$

5.1 $D = t^2 - 4\cdot 2\cdot 2 = t^2 - 16 = (t+4)(t-4)$; $\underline{N_{1/2}\left(\frac{1}{4}\left(-t \pm \sqrt{t^2-16}\right) \middle| 0\right)}$

5.2 $t_1 = -4 \;;\; t_2 = 4 \;;\; N_1(1|0) \;;\; N_2(-1|0) \leftarrow t = 4$

Für F.u: siehe 2c)

5.3 $f_5(x) = 2x^2 + 5x + 2 = 2\left(x^2 + \frac{5}{2}x + 1\right)$

$N_{1/2}\left(\frac{1}{4}\left(-5 \pm \sqrt{25-16}\right) \middle| 0\right) = N_{1/2}\left(\frac{1}{4}(-5 \pm 3) \middle| 0\right)$

$x_1 = -2 \;;\; x_2 = -\frac{1}{2}$

$\underline{f_5(x) = 2(x+2)\left(x+\frac{1}{2}\right)}$

$\frac{1}{2}\cdot 2 = 1$

$\frac{1}{2} + 2 = \frac{5}{2}$

5.4 $P(\underset{x}{1}|\underset{y}{2})$ in $f_t(x)$:

$2\cdot 1^2 + 1\cdot t + 2 = 2 \Leftrightarrow \underline{t = -2} \;;\; f_{-2}(x) = 2x^2 - 2x + 2$

5.5 $f_{-2}(x) = h(x)$

$\Leftrightarrow 2x^2 - 2x + 2 = \frac{1}{2}x + 3/2 \quad \Leftrightarrow 2x^2 - \frac{5}{2}x + \frac{1}{2} = 0 \quad | \cdot 2$

$\Leftrightarrow 4x^2 - 5x + 1 = 0$

$x_{1/2} = \frac{1}{8}\left(5 \pm \sqrt{25 - 4\cdot 4\cdot 1}\right) = \frac{1}{8}(5 \pm 3)$

$x_1 = \frac{1}{4} \;;\; h\left(\frac{1}{4}\right) = \frac{13}{8} \Rightarrow \underline{SP_1\left(\frac{1}{4} \middle| \frac{13}{8}\right)}$

$x_2 = 1 \;;\; h(1) = 2 \Rightarrow \underline{SP_2(1|2)}$ (= P von 5.4)

5.6 $f_5(x) = g_r(x)$

$\Leftrightarrow 2x^2 + 5x + 2 = -3x + r$

$\Leftrightarrow 2x^2 + 8x + 2 - r = 0$

$D = 64 - 4\cdot 2(2-r) =$

$= 64 - 16 + 8r$

$= 48 + 8r$

$1\ SP \Leftrightarrow D = 0$

$48 + 8r = 0$

$\Leftrightarrow \underline{r = -6}$

$D = 0$

$x_{1/2} = -\frac{b}{2a} = -\frac{8}{2\cdot 2} = -2$

$g_{-6}(-2) = -3\cdot(-2) - 6 = 0$

$\underline{SP(-2|0)}$ (= N_1 von 5.3)